

Asymmetric nuclear matter and neutron star properties

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In this work we calculate the total mass, radius, moment of inertia, and surface gravitational redshift for neutron stars using various equations of state (EOS). Modern meson-exchange potential models are used to evaluate the G -matrix for asymmetric nuclear matter. We calculate both a non-relativistic and a relativistic EOS. Of importance here is the fact that relativistic Brueckner-Hartree-Fock calculations for symmetric nuclear matter fit the empirical data, which are not reproduced by non-relativistic calculations. Relativistic effects are known to be important at high densities, giving an increased repulsion. This leads to a stiffer EOS compared to the EOS derived with a non-relativistic approach. Both the non-relativistic and the relativistic EOS yield moments of inertia and redshifts in agreement with the accepted values. The relativistic EOS yields, however, too large mass and radius. The implications are discussed.

I. INTRODUCTION

The physics of compact objects like neutron stars offers an intriguing interplay between nuclear processes and astrophysical observables (Pethick & Ravenhall 1992; Weber & Glendenning 1993). Neutron stars exhibit conditions far from those encountered on earth; typically, expected densities ρ of a neutron star interior are of the order of 10^3 or more times the density $\rho_d \approx 4 \cdot 10^{11}$ g/cm³ at neutron “drip”. Thus, the determination of an equation of state (EOS) for dense matter is essential to calculations of neutron star properties; the EOS determines properties such as the mass range, the mass-radius relationship, the crust thickness and the cooling rate (Weber & Glendenning 1993; Pethick et al. 1995; Lorenz et al. 1993). The same EOS is also crucial in calculating the energy released in a supernova explosion. Clearly, the relevant degrees of freedom will not be the same in the crust, where the density is much smaller than the nuclear matter saturation density, and in the center of the star where the density is so high that models based solely on interacting nucleons are questionable. Data from neutron stars indicate that the EOS should probably be moderately stiff in order to support maximum neutron star masses in a range from approximately $1.4M_\odot$ to $1.9M_\odot$ (Thorsett et al. 93),

where M_\odot is the solar mass. In addition, simulations of supernova explosions seem to require an EOS which is even softer. A combined analysis of the data coming from binary pulsar systems and from neutron star formation scenarios is done by Finn (1994), where it is shown that neutron star masses should fall predominantly in the range $1.3 \leq M/M_\odot \leq 1.6$. In addition, a theoretical result for the maximum mass of neutron stars will have very important astrophysical implications for the existence and number of black holes in the universe; examples are the famous galactic black hole candidates Cyg X-1 (Gies & Bolton 1986) and LMC X-3 (Cowley et al. 1983) which are massive X-ray binaries. Their mass functions ($0.25 M_\odot$ and $2.3 M_\odot$) are, however, smaller than for some low-mass X-ray binaries like A0620-00 (Mc Clintock & Remillard 1986) and V404 Cyg (Casares et al. 1992), which make even better black hole candidates with mass functions in excess of three solar masses.

Several theoretical approaches to calculations of the EOS have been considered. The hypothesis that strange quark matter may be the absolute ground state of the strong interaction (Witten 1984), has been used by Glendenning (1991) and Rosenhauer et al. (1992) in the investigation of the possibility of interpreting pulsars as rotating strange stars. Other approaches introduce exotic states of nuclear matter, such as kaon (Brown 1994; Thorsson et al. 1994; Brown et al. 1994; Muto et al. 1993) or pion condensation (Migdal et al. 1990; Takatsuka et al. 1993; Mittet & Østgaard 1988).

The scope of this work is to derive the EOS from the underlying many-body theory, derived from a realistic nucleon-nucleon (NN) interaction. From this EOS we will study neutron-star observables such as mass-radius relationship, surface gravitational redshift and moment of inertia. By realistic we shall mean a nucleon-nucleon interaction defined within the framework of meson-exchange theory, described conventionally in terms of one-boson-exchange (OBE) models (Machleidt 1989; Machleidt & Li 1993). Explicitly, we will here build on the Bonn meson-exchange potential models as they are defined by Machleidt (1989) in table A.2. Furthermore, the physically motivated coupling constants and energy cut-offs which determine the OBE potentials are constrained through a fit to the available scattering data. The subsequent step is to obtain an effective NN interaction in the nuclear medium by solving the Bethe-Goldstone equation self-consistently. Thus, the only parameters which enter the theory are those which define the NN potential. Until recently, most microscopic calculations of the EOS for nuclear matter have been carried out within a non-relativistic framework (Wiringa et al. 1988), where the non-relativistic Schrödinger equation is used to describe the single-particle motion in the nuclear medium. Various degrees of sophistication are accounted for in the literature (Pethick & Ravenhall 1992; Machleidt 1989) ranging from first-order calculations in the reaction matrix G to the inclusion of two- and three-body higher-order effects (Machleidt 1989; Wiringa et al. 1988; Dickhoff & Muther 1992; Kuo & Ma 1983; Kuo et al. 1987).

A common problem to non-relativistic nuclear matter calculations is, however, the simultaneous reproduction of both the binding energy per nucleon ($BE/A = -16 \pm 1$ MeV) and the saturation density with a Fermi momentum $k_F = 1.35 \pm 0.05$ fm $^{-1}$. Results obtained with a variety of methods and nucleon-nucleon (NN) interactions are located along a band denoted the “Coester band”, which does not satisfy the empirical data for nuclear matter. Albeit these deficiencies, much progress has been achieved recently in the description of the saturation properties of nuclear matter. Of special relevance is the replacement of the non-relativistic Schrödinger equation with the Dirac equation to describe the single-particle motion, referred to as the Dirac-Brueckner (DB) approach. This is motivated by the success of the phenomenological Dirac approach in nucleon-nucleus scattering (Ray et al. 1991) and in the description of properties of finite nuclei (Nikolaus et al. 1992), such as the spin-orbit splitting in finite nuclei (Brockmann 1978). Moreover, rather promising results within the framework of the DB approach have been obtained (Brockmann & Machleidt 1990; Li et al. 1992; Muther et al. 1990), employing the OBE models of the Bonn group. Actually, the empirical properties of nuclear matter are quantitatively reproduced by Brockmann & Machleidt (1990).

Further, another scope of this work is to derive the EOS for asymmetric nuclear matter, using both the non-relativistic Brueckner-Hartree-Fock (BHF) approach and the relativistic extension of this, the Dirac-Brueckner-Hartree-Fock (DBHF) approach (Brockmann & Machleidt 1990; Celenza & Shakin 1986). A preliminary discussion of these results has been presented recently (Engvik et al. 1994).

We will treat the Pauli operator, which enters our formalism (see below), correctly. Asymmetric nuclear matter is important in e.g. studies of neutron star cooling, as demonstrated recently by Lattimer et al. (1991). They showed that ordinary nuclear matter with a small asymmetry parameter can cool by the so-called direct URCA process even more rapidly than matter in an exotic state.

However, as discussed above, an EOS with only nucleonic degrees of freedom may not be too realistic in the crust region and at high densities in the interior of the star. Thus, in our study of neutron star observables we will have to link our EOS with equations of state which take into account degrees of freedom other than the nucleonic ones.

This work falls in four sections. After the introductory remarks, we briefly review the general picture in section 2. In this section we also recast some of the astrophysical equations, together with the equations of state derived within both the non-relativistic and the Dirac-Brueckner-Hartree-Fock approaches. The results are presented in section 3, while discussions and conclusions are given in section 4.

II. GENERAL THEORY

In the interior of neutron stars, we find matter at densities above the neutron “drip” $\rho_d \approx 4 \cdot 10^{11}$ g/cm 3 , the density at which nuclei begin to dissolve and merge together, and the properties of cold dense matter and the associated equation of state are reasonably well understood at densities up to $\rho_n \approx 3 \cdot 10^{14}$ g/cm 3 . In the high-density range above ρ_n the physical properties of matter are still uncertain.

In the region between ρ_d and ρ_n matter is composed mainly of nuclei, electrons and free neutrons. The nuclei disappear at the upper end of this density range because their binding energy decreases with increasing density. The nuclei then become more neutron-rich and their stability decreases until a critical value of the neutron number is reached, at which point the nuclei dissolve, essentially by merging together. Since the nuclei present are very neutron-rich, the matter inside nuclei is very similar to the free neutron gas outside. However, the external neutron gas reduces the nuclear surface energy appreciably, and it must vanish when the matter inside nuclei becomes identical to that outside.

The free neutrons supply an increasingly larger fraction of the total pressure as the density increases, but at neutron drip the pressure is almost entirely due to electrons. Slightly above neutron drip the adiabatic index drops sharply since the low-density neutron gas contributes appreciably to the density but not much to the pressure, and it does not rise again above $4/3$ until $\rho > 7 \cdot 10^{12} \text{ g/cm}^3$. This means that no stable stars can have central densities in this region.

Relatively "soft" equations of state have been proposed since the average system energy is attractive at nuclear densities. However, "stiff" equations of state may result from potentials for which the average system interaction energy is dominated by the attractive part of the potential at nuclear densities, but by the repulsive part at higher densities. The stiffer equations of state give rise to important changes in the structure and masses of heavy neutron stars. As the interaction energy becomes repulsive above nuclear densities, the corresponding pressure forces are better able to support stellar matter against gravitational collapse. The result is that the maximum masses of stars based on stiff equations of state are greater than those based on soft equations of state. Also, stellar models based on stiffer equations of state have a lower central density, a larger radius and a thicker crust. Such differences are important in determining mass limits for neutron stars, their surface properties, moments of inertia, etc.

For low densities $\rho < \rho_n$, where the nuclear force is expected to be attractive, the pressure is softened somewhat by the inclusion of interactions. For very high densities, however, the equation of state is hardened due to the dominance of the repulsive core in the nuclear potential.

At very high densities above 10^{15} g/cm^3 , the composition is expected to include an appreciable number of hyperons and the nucleon interactions must be treated relativistically. Relativistic many-body techniques for strongly interacting matter are, however, not fully developed. Presently developed nuclear equations of state are also subject to many uncertainties, such as the possibility of neutron and proton superfluidity, of pion or kaon condensation, of neutron solidification, of phase transition to "quark matter", and the consequences of the Δ -nucleon resonance.

At densities significantly greater than ρ_n , it is no longer possible to describe nuclear matter in terms of a non-relativistic many-body Schrödinger equation. The "meson clouds" surrounding the nucleons begin to overlap and the picture of localized individual particles interacting via two-body forces becomes invalid. Even before this "break-down" different potentials which reproduce reliably low-energy phase shift data result in different equations of state, since the potentials are sensitive to the repulsive core region unaffected by the phase-shift data. If the fundamental building blocks of all strongly interacting particles are quarks, any description of nuclear matter at very high densities should involve quarks. When nuclei begin to "touch", matter just above this density should undergo a phase transition at which quarks would begin to "drip" out of the nucleons and the result would be quark matter, a degenerate Fermi liquid.

The main uncertainty in neutron star models is the equation of state of nuclear matter, particularly above typical nuclear densities of $\rho \sim 2.8 \cdot 10^{14} \text{ g/cm}^3$. But our present understanding of the condensed matter is already sufficient to place quite strict limits on masses and radii of stable neutron stars.

Neutron star models including realistic equations of state give the following general results: Stars calculated with a stiff equation of state have a lower central density, a larger radius, and a much thicker crust than stars of the same mass computed from a soft equation of state. Pion or kaon condensation and quark matter would tend to contract neutron stars of a given mass and decrease the maximum mass.

Calculations give the following configurations in the interior: The surface for $\rho < 10^6 \text{ g/cm}^3$ is a region in which temperatures and magnetic fields may affect the equation of state. The outer crust for $10^6 \text{ g/cm}^3 < \rho < 4 \cdot 10^{11} \text{ g/cm}^3$ is a solid region where a Coulomb lattice of heavy nuclei coexist in β -equilibrium with a relativistic degenerate electron gas. The inner crust for $4 \cdot 10^{11} \text{ g/cm}^3 < \rho < 2 \cdot 10^{14} \text{ g/cm}^3$ consists of a lattice of neutron-rich nuclei together with a superfluid neutron gas and an electron gas. The neutron liquid for $2 \cdot 10^{14} \text{ g/cm}^3 < \rho < 8 \cdot 10^{14} \text{ g/cm}^3$ contains mainly superfluid neutrons with a smaller concentration of superfluid protons and normal electrons. The core region for $\rho > 8 \cdot 10^{14} \text{ g/cm}^3$ may not exist in some stars and will depend on the existence of pion or kaon condensation, neutron solid, quark matter, etc.

The minimum mass of a stable neutron star can be determined from a minimum in the mass-radius relation $M(R)$, or by setting the mean value of the adiabatic index Γ equal to the critical value for radial stability against collapse. The resulting minimum mass is $M \sim 0.1M_\odot$, where M_\odot is the solar mass, with a corresponding central density of $\rho \sim 10^{14} \text{ g/cm}^3$ and radius $R \sim 200 \text{ km}$. The maximum mass equilibrium configuration is somewhat uncertain, but all microscopic calculations give $M < 2.7M_\odot$. Note that the adiabatic index criterion for the minimum mass is an approximate one, while the one involving the $M(R)$ condition is a precise one, if the mass limit is due to instability of a radial mode.

Astronomical observations leading to global neutron star parameters such as total mass, radius, or moment of inertia, are important since they are sensitive to microscopic model calculations. The most reliable way of determining masses is via Kepler's third law in binary pulsars. Observations of such pulsars give (approximately) a common mass region consistent with all data of $1.3M_\odot < M < 1.9M_\odot$. Present mass determinations for neutron stars are all consistent with present stellar evolution theories. There is, however, no firm evidence yet about the value for the maximum

mass of a neutron star. A general limit for the maximum mass can be estimated by assuming the following: General relativity is the correct theory of gravitation. The equation of state must satisfy the "microscopic stability" condition $dP/d\rho \geq 0$ and the causality condition $dP/d\rho < c^2$, and should match some known low-density equation of state. This gives an upper limit of $M \sim 3 - 5M_\odot$. Stiff equations of state in calculations predict a maximum mass in the range $M \sim 1.5 - 2.7M_\odot$. Rotation may increase the maximum neutron star mass, but not appreciably, i.e., $< 20\%$.

We aim here at discriminating between equations of state for asymmetric nuclear matter derived from non-relativistic and relativistic approaches (to be discussed below). As discussed above, relativistic effects become important at densities higher than ρ_n , and it is therefore of interest to understand whether the two approaches yield significantly different neutron star properties. The derivation of the equations of state is discussed in the first subsection, whereas the equations which define the calculations of mass, radius, moment of inertia and gravitational redshifts are discussed in the subsequent subsection.

A. Derivation of the equation of state for asymmetric nuclear matter

In this subsection we discuss both the non-relativistic and the relativistic approach to the EOS in the framework of the Brueckner-Hartree-Fock (BHF) theory.

1. Brueckner-Hartree-Fock approach for asymmetric nuclear matter

Following the conventional many-body approach, we divide the full hamiltonian $H = T + V$, with T being the kinetic energy and V the bare NN potential, into an unperturbed part $H_0 = T + U$ and an interacting part $H_I = V - U$, such that

$$H = T + V = H_0 + H_I,$$

where we have introduced an auxiliary single-particle (sp) potential U . If U is chosen such that H_I becomes small, then perturbative many-body techniques can presumably be applied. A serious obstacle to any perturbative treatment is the fact that the bare NN potential V is very large at short inter-nucleonic distances, which renders a perturbative approach highly prohibitive. To overcome this problem, we introduce the reaction matrix G given by the solution of the Bethe-Goldstone equation (in operator form)

$$G(E) = V + VQ \frac{1}{E - QH_0Q} QG, \quad (1)$$

where E is the energy of the interacting nucleons and Q is the the Pauli operator which prevents scattering into occupied states. The Pauli operator is given by

$$Q(k_m\tau_m, k_n\tau_n) = \begin{cases} 1, & k_m > k_F^{\tau_m}, k_n > k_F^{\tau_n}, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

in the laboratory system, where $k_F^{\tau_i}$ defines the Fermi momenta of the proton ($\tau_i = 1/2$) and neutron ($\tau_i = -1/2$). For notational economy, we set $|\mathbf{k}_m| = k_m$.

The above expression for the Pauli operator is in the laboratory frame. In the calculations of the G -matrix, we will employ a Pauli operator in the center of mass and relative coordinate system. Further, this Pauli operator will be given by the so-called angle-average approximation, for details see Bao et al. (1994). Eq. (1) reads then (in a partial wave representation)

$$G_{ll'}^{\alpha T_z}(kk'KE) = V_{ll'}^{\alpha T_z}(kk') + \sum_{l''} \int \frac{d^3q}{(2\pi)^3} V_{ll''}^{\alpha T_z}(kq) \frac{Q^{T_z}(q,K)}{E - H_0} G_{l''l'}^{\alpha T_z}(qk'KE), \quad (3)$$

with ll' and kk' the orbital angular momentum and the linear momentum of the relative motion, respectively. The angle-average Pauli operator is given by Q^{T_z} , where T_z is the total isospin projection. Further K is the momentum of the center-of-mass motion. Since we are going to use an angular average for the Pauli operator, the G -matrix is diagonal in total angular momentum J . Further, the G -matrix is diagonal in the center-of-mass orbital momentum L and the total spin S . These quantities, i.e. J , L and S , are all represented by the variable α . Three different G -matrices have to be evaluated, depending of the individual isospins ($\tau_1\tau_2$) of the interacting nucleons ($\frac{1}{2}\frac{1}{2}$, $-\frac{1}{2} - \frac{1}{2}$

and $-\frac{1}{2}\frac{1}{2}$). These quantities are represented by the total isospin projection T_z in eq. (3). The different G-matrices originate from the discrimination between protons and neutrons in eq. (2). The term H_0 in the denominator of eq. (3) is the unperturbed energy of the intermediate states and depends on k , K and the individual isospin of the interacting particles; further discussion is given below.

We use a continuous single particle (sp) spectrum advocated by Mahaux et al. (1989). It is defined by the self-consistent solution of the following equations¹:

$$\varepsilon_i = t_i + u_i = \frac{k_i^2}{2m} + u_i, \quad (4)$$

where m is the bare nucleon mass, and

$$u_i = \sum_{h \leq k_F} \langle ih | G(E = \varepsilon_i + \varepsilon_h) | ih \rangle_{AS}. \quad (5)$$

In eqs. (4)-(5), the subscripts i and h represent the quantum numbers of the single-particle states, such as isospin projections τ_i and τ_h , momenta k_i and k_h etc. The sp kinetic energy is given by t_i and similarly the sp potential by u_i . For further details see Bao et al. 1994.

Finally, the non-relativistic energy per particle \mathcal{E}/A is formally given as

$$\begin{aligned} \mathcal{E}/A = & \frac{1}{A} \sum_{h \leq k_F} \frac{k_h^2}{2m} + \\ & \frac{1}{2A} \sum_{h \leq k_F, h' \leq k_F} \langle hh' | G(E = \varepsilon_h + \varepsilon_{h'}) | hh' \rangle_{AS} \end{aligned} \quad (6)$$

In this equation we have suppressed the isospin indices for the Fermi momenta.

2. Relativistic effects

The properties of neutron stars depend on the equation of state at densities up to an order of magnitude higher than those observed in ordinary nuclei. At such densities, relativistic effects certainly prevail. Among relativistic approaches to the nuclear many-body problem, the so-called Dirac-Hartree and Dirac-Hartree-Fock approaches have received much interest (Serot & Walecka 1986; Serot 1992; Horowitz & Serot 1987). One of the early successes of these approaches was the quantitative reproduction of spin observables, which are only poorly described by the non-relativistic theory. Important to these methods was the introduction of a strongly attractive scalar component and a repulsive vector component in the nucleon self-energy (Brockmann 1978; Serot & Walecka 1986). Inspired by the successes of the Dirac-Hartree-Fock method, a relativistic extension of Brueckner theory was proposed by Celenza & Shakin (1986), known as the Dirac-Brueckner theory. One of the appealing features of the Dirac-Brueckner approach is the self-consistent determination of the relativistic sp energies and wave functions. The Dirac-Brueckner approach differs from the Dirac-Hartree-Fock one in the sense that in the former one starts from the free NN potential which is only constrained by a fit to the NN data, whereas the Dirac-Hartree-Fock method pursues a line where the parameters of the theory are determined so as to reproduce the bulk properties of nuclear matter. It ought, however, to be stressed that the Dirac-Brueckner approach, which starts from NN potentials based on meson exchange, is a non-renormalizable theory where short-range part of the potential depends on additional parameters like vertex cut-offs, clearly minimizing the sensitivity of calculated results to short-distance inputs (Brockmann & Machleidt 1990; Celenza & Shakin 1986; ter Haar & Malfliet 1987). The description presented here for the Dirac-Brueckner approach follows closely that of Brockmann & Machleidt (1990). We will thus use the meson-exchange models of the Bonn group, defined in table A.2, Machleidt (1989). There the three-dimensional reduction of the Bethe-Salpeter equation as given by the Thompson equation is used to solve the equation for the scattering matrix (Thompson 1970). Hence, including the necessary medium effects like the Pauli operators discussed in the previous subsection and the starting energy, we will rewrite eq. (3) departing from the Thompson equation. Then, in a self-consistent way, we determine the above-mentioned scalar and vector components which define the nucleon self-energy.

¹ In this work we will throughout set $G = c = \hbar = 1$, where G is the gravitational constant.

In order to introduce the relativistic nomenclature, we consider first the Dirac equation for a free nucleon, i.e.,

$$(i \not{\partial} - m)\psi(x) = 0,$$

where m is the free nucleon mass and $\psi(x)$ is the nucleon field operator (x is a four-point) which is conventionally expanded in terms of plane wave states and the Dirac spinors $u(p, s)$, and $v(p, s)$, where $p = (p^0, \mathbf{p})$ is a four momentum² and s is the spin projection.

The positive energy Dirac spinors are (with $\bar{u}u = 1$)

$$u(p, s) = \sqrt{\frac{E(p) + m}{2m}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E(p) + m} \chi_s \end{pmatrix}, \quad (7)$$

where χ_s is the Pauli spinor and $E(p) = \sqrt{m^2 + |\mathbf{p}|^2}$. To account for medium modifications to the free Dirac equation, we introduce the notion of the self-energy $\Sigma(p)$. As we assume parity to be a good quantum number, the self-energy of a nucleon can be formally written as

$$\Sigma(p) = \Sigma_S(p) - \gamma_0 \Sigma^0(p) + \boldsymbol{\gamma} \mathbf{p} \Sigma^V(p).$$

The momentum dependence of Σ^0 and Σ_S is rather weak (Serot & Walecka 1986). Moreover, $\Sigma^V \ll 1$, such that the features of the Dirac-Brueckner-Hartree-Fock procedure can be discussed within the framework of the phenomenological Dirac-Hartree ansatz, i.e. we approximate

$$\Sigma \approx \Sigma_S - \gamma_0 \Sigma^0 = U_S + U_V,$$

where U_S is an attractive scalar field and U_V is the time-like component of a repulsive vector field. The finite self-energy modifies the free Dirac spinors of eq. (7) as

$$\tilde{u}(p, s) = \sqrt{\frac{\tilde{E}(p) + \tilde{m}}{2\tilde{m}}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{\tilde{E}(p) + \tilde{m}} \chi_s \end{pmatrix},$$

where we let the terms with tilde represent the medium modified quantities. Here we have defined

$$\tilde{m} = m + U_S,$$

and

$$\tilde{E}_i = \tilde{E}(p_i) = \sqrt{\tilde{m}_i^2 + \mathbf{p}_i^2}. \quad (8)$$

As in the previous subsection, the subscripts i , and h below, represent the quantum numbers of the single-particle states, such as isospin projections τ_i and τ_h , momenta k_i and k_h , etc.

The sp energy is

$$\tilde{\epsilon}_i = \tilde{E}_i + U_V^i, \quad (9)$$

and the sp potential is given by the G-matrix as

$$u_i = \sum_{h \leq k_F} \frac{\tilde{m}_i \tilde{m}_h}{\tilde{E}_h \tilde{E}_i} \langle ih | \tilde{G}(\tilde{E} = \tilde{E}_i + \tilde{E}_h) | ih \rangle_{AS}, \quad (10)$$

or, if we wish to express it in terms of the constants U_S and U_V , we have

$$u_i = \frac{\tilde{m}_i}{\tilde{E}_i} U_S^i + U_V^i. \quad (11)$$

²Further notation is as given in Itzykson Zuber (1980).

In eq. (10), we have introduced the relativistic \tilde{G} -matrix. If the two interacting particles, with isospins τ_1 and τ_2 , yield a total isospin projection T_z , the relativistic \tilde{G} -matrix in a partial wave representation is given by

$$\tilde{G}_{l'l'}^{\alpha T_z}(kk'K\tilde{E}) = \tilde{V}_{l'l'}^{\alpha T_z}(kk') + \sum_{l''} \int \frac{d^3q}{(2\pi)^3} \tilde{V}_{l''}^{\alpha T_z}(kq) \frac{\tilde{m}_1 \tilde{m}_2}{\tilde{E}_1^q \tilde{E}_2^q} \frac{Q^{T_z}(q, K)}{(\tilde{E} - \tilde{E}_1^q - \tilde{E}_2^q)} \tilde{G}_{l''l'}^{\alpha T_z}(qk'K\tilde{E}), \quad (12)$$

where the relativistic starting energy is defined according to eq. (8) as

$$\tilde{E} = \tilde{E}(\sqrt{k^2 + K^2/4}, \tau_1) + \tilde{E}(\sqrt{k^2 + K^2/4}, \tau_2).$$

and

$$\tilde{E}_{1(2)}^q = \tilde{E}(\sqrt{q^2 + K^2/4}, \tau_{1(2)})$$

Equations (9)-(12) are solved self-consistently, starting with adequate values for the scalar and vector components U_S and U_V . This iterative scheme is continued until these parameters show little variation. The calculations are carried out in the neutron matter rest frame, avoiding thereby a cumbersome transformation between the two-nucleon center-of-mass system and the neutron matter rest frame.

Finally, the relativistic version of eq. (??) reads

$$\begin{aligned} \mathcal{E}/A = & \frac{1}{A} \sum_{h \leq k_F} \frac{\tilde{m}_h m + k_h^2}{\tilde{E}_h} + \\ & \frac{1}{2} \sum_{h \leq k_F, h' \leq k_F} \frac{\tilde{m}_h \tilde{m}_{h'}}{\tilde{E}_h \tilde{E}_{h'}} \langle hh' | \tilde{G}(\tilde{E} = \tilde{E}_h + \tilde{E}_{h'}) | hh' \rangle_{AS} - m. \end{aligned} \quad (13)$$

B. Neutron star equations

We end this section by presenting the formalism needed in order to calculate the mass, radius, moment of inertia and gravitational redshift. We will assume that the neutron stars we study exhibit an isotropic mass distribution. Hence, from the general theory of relativity, the structure of a neutron star is determined through the Tolman-Oppenheimer-Volkov eqs., i.e.,

$$\frac{dP}{dr} = - \frac{\{\rho(r) + P(r)\} \{M(r) + 4\pi r^3 P(r)\}}{r^2 - 2rM(r)}, \quad (14)$$

and

$$\frac{dM}{dr} = 4\pi r^2 \rho(r), \quad (15)$$

where $P(r)$ is the pressure, $M(r)$ is the gravitational mass inside a radius r , and $\rho(r)$ is the mass-energy density. The latter equation is conventionally written as an integral equation

$$M(r) = 4\pi \int_0^r \rho(r') r'^2 dr'. \quad (16)$$

In addition, the main ingredient in a calculation of astrophysical observables is the equation of state (EOS)

$$P(n) = n^2 \left(\frac{\partial \epsilon}{\partial n} \right), \quad (17)$$

where $\epsilon = \mathcal{E}/A$ is the energy per particle and n is the particle number density. Eqs. (14), (16) and (17) are the basic ingredients in our calculations of neutron star properties.

The moment of inertia I for a slowly rotating symmetric neutron star is related to the angular momentum J and the angular velocity Ω in an inertial system at infinity through

$$I = \left(\frac{\partial J}{\partial \Omega} \right)_{\Omega=0} = \frac{J}{\Omega}. \quad (18)$$

The angular momentum J is given by

$$J = \frac{1}{6} R^4 \left(\frac{d\tilde{\omega}}{dr} \right)_{r=R}, \quad (19)$$

where $\tilde{\omega}$ is the angular velocity relative to particles with zero angular momentum. Further, the angular velocity Ω is

$$\Omega = \tilde{\omega}(R) + \frac{u(R)}{3R^3}, \quad (20)$$

with

$$u = r^4 \frac{d\tilde{\omega}}{dr}. \quad (21)$$

Finally, the gravitational redshift Z_s is given by

$$Z_s = \left(1 - \frac{2M(R)}{R} \right)^{-1/2} - 1. \quad (22)$$

To calculate the total mass, radius, moment of inertia and gravitational redshift, we employ the EOS defined in eq. (17) with the boundary conditions

$$P_c = P(n_c), \quad M(0) = 0,$$

where we let the subscript c refer to the center of the star, and n_c is the central density which is our input parameter in the calculations of neutron star properties.

III. RESULTS

A. The equation of state

As mentioned in the introduction, the replacement of the non-relativistic Schrödinger equation by the Dirac equation offers a quantitative reproduction of the saturation properties of nuclear matter. Central to these results is the use of modern meson-exchange potentials with a weak tensor force, where the strength of the tensor force is reflected in the D -state probability of the deuteron. The main differences in the strength of the tensor force in nuclear matter arises in the $T = 0$ 3S_1 - 3D_1 channel, though other partial waves also give rise to tensor force contributions. To derive the equation of state, we start from the Bonn NN potential models as they are defined by the parameters of table A.2, Machleidt (1989). These potentials are recognized by the labels A, B and C, with the former carrying the weakest tensor force. Since all three potentials have to reproduce the same set of scattering data, a potential containing a weak tensor component needs a strong central component. Due to the fact that the tensor force is more quenched than the central force in a medium, the three potentials have different off-shell properties. The stronger off-shell interaction of version A give rise to more binding energy in symmetric nuclear matter.

In neutron matter ($T = 1$), however, the important 3S_1 - 3D_1 channel does not contribute to the energy per particle, and the difference between the various potentials is found to be small. This is indeed the case, as reported in neutron matter calculations (Li et al. 1992; Bao et al. 1994). As the proton fraction increases, the $T = 0$ contribution becomes more important and the difference between the potentials becomes significant. In our presentation we have used version A, since this potential reproduces the empirical properties of nuclear matter in the Dirac-Brueckner approach. The energy shift going from symmetric nuclear matter to pure neutron matter is largest for potential A, since A has the strongest off-shell $T = 0$ interaction.

In fig. 1 we present the results for both relativistic and non-relativistic calculations. The relativistic effects become significant at normal nuclear matter densities. The relativistic calculations give an increased repulsion at higher densities and correspondingly stiffer EOS than the non-relativistic approaches.

Independent of the proton fraction the relativistic effects become significant at about 0.15 fm^{-3} . Moreover, the energy shift due to the relativistic effects is almost the same for all different proton fractions. The energy shift is only a few MeV larger for neutron matter than for symmetric nuclear matter.

The differences between the relativistic and non-relativistic results can be understood from the following two arguments, see e.g. Brown et al. (1987). Firstly, relativistic effects introduce a strongly density dependent repulsive term in the energy per particle, of the order of $(n/n_0)^{8/3}$, where n_0 is the nuclear matter saturation density in fm^{-3} . This contribution³ is important in order to saturate nuclear matter, and is interpreted by Brown et al. (1987) as a density dependent correction to the mass of the scalar boson σ , which is responsible for the scalar term U_S in the relativistic nucleon mass. In the vacuum, the σ -meson has self-energy contributions due to its coupling to virtual nucleon-antinucleon pairs. In nuclear matter, scattering into states with $k < k_F$ are Pauli blocked, giving in turn a repulsive contribution to the energy per particle. Secondly, the nucleon-nucleon spin-orbit interaction is enhanced (the spin-orbit force is repulsive), since the relativistic effective mass is changed due to the scalar fields which couple to negative energy states. For further details on the relativistic effects, see the discussion in chapter 10 of Machleidt (1989).

For relatively small proton fractions, the energy per particle exhibits much the same curvature as the curve for pure neutron matter at high densities, although the energy per particle is less repulsive at high densities. At lower densities, the situation is rather different. This is due to the contributions from various isospin $T = 0$ partial waves, especially the contribution from the 3S_1 - 3D_1 channel, where the tensor force component of the nucleon-nucleon potential provides additional binding. From fig. 1 we note that with a proton fraction of 15%, the energy per particle starts to become attractive at low densities (in the region 0.07 fm^{-3} to 0.3 fm^{-3}). For larger proton fractions, the attraction is increased.

Using the non-relativistic and the relativistic equations of state, we wish to study how sensitive various neutron star properties are with respect to different proton fractions. Note also that within the Dirac-Brueckner approach, the Bonn A potential reproduces the empirical nuclear matter binding energy and saturation density (Brockmann & Machleidt 1990). This gives a more consistent approach to asymmetric nuclear matter. The reader should, however, keep in mind that there are several mechanisms (to be discussed in section 4) which may reduce the stiffness of the above equations of state.

B. Mass, radius, moment of inertia and surface gravitational redshift

To calculate mass, radius, moment of inertia and surface gravitational redshift we need the EOS for all relevant densities. The equations of state derived in the previous subsection have a limited range, $0.1 \text{ fm}^{-3} \leq n \leq 0.8 \text{ fm}^{-3}$. We must therefore include equations of state for other densities as well. These equations of state are discussed below.

For the lowest densities, we use the equation of state by Haensel, Zdunik and Dobaczewski (1989). This equation of state (HZD) is obtained in the following way: The pressure is fitted by a polynomial consisting of 9 terms, i.e.,

$$P(X) = \sum_{i=1}^9 C_i X^{l_i}, \quad (23)$$

where

$$X = 1.6749 \times 10^5 n, \quad (24)$$

n is given in $[\text{fm}^{-3}]$, and the values

$$n = 0.077, \quad 0.154, \quad 0.395, \quad 0.762, \quad 1.575, 3.147, \quad 6.443, \quad 12.240, \quad 26.551,$$

in $[10^{-5}/\text{fm}^3]$, are chosen to give the coefficients C_i . The corresponding equations are solved by matrix inversion, and we obtain

$$\begin{aligned} P(X) = & 8.471521942X^{1/3} - 40.437728191X^{2/3} + 74.927783479X \\ & - 67.102601796X^{4/3} + 30.011422630X^{5/3} - 4.207322319X^2 \\ & - 1.419954871X^{7/3} + 0.589441363X^{8/3} - 0.060468689X^3, \end{aligned} \quad (25)$$

³The relativistic effects can also be understood as a special class of many-body forces.

where $P(X)$ is given in units of $[10^{27} \text{ N/m}^2]$ and in the density range of $2 \times 10^{-6} \text{ fm}^{-3} < n < 2.84 \times 10^{-4} \text{ fm}^{-3}$. We need all the decimals in the different terms to get an accuracy of at least two decimals in the net equation.

The Baym-Bethe-Pethick (BBP) equations of state (Baym et al. 1971) are taken from Øvergård and Østgaard (1991). The given data are fitted by two five-term polynomials to give (BBP-1)

$$P(n) = 4.3591n^{4/3} - 122.4841n^{5/3} + 1315.2746n^2 - 6180.0702n^{7/3} + 10659.0049n^{8/3}, \quad (26)$$

where $P(n)$ is given in units of $[10^{34} \text{ N/m}^2]$ for n in the density range of $0.00027 \text{ fm}^{-3} < n < 0.0089 \text{ fm}^{-3}$, and (BBP-2)

$$P(n) = 0.092718n^{4/3} - 0.035382n^{5/3} + 1.193525n^2 - 2.424555n^{7/3} + 2.472867n^{8/3}, \quad (27)$$

where $P(n)$ is given in units of $[10^{34} \text{ N/m}^2]$ for n in the density range of $0.0089 \text{ fm}^{-3} < n < 0.3 \text{ fm}^{-3}$.

The Arntsen- Øvergård (AØ-5) equation of state is given by a five-term polynomial (Øvergård & Østgaard 1991), i.e.,

$$P(n) = 9.4433n^{5/3} - 34.6909n^2 + 102.6575n^{8/3} - 87.6158n^3 + 14.3549n^{11/3}, \quad (28)$$

where $P(n)$ is given in units of $[10^{34} \text{ N/m}^2]$ for n in the density range of $0.4 \text{ fm}^{-3} < n < 3.6 \text{ fm}^{-3}$.

The equation of state by Pandharipande & Smith (1975) (PS) is taken from Øvergård and Østgaard (1991). The given data are fitted by a five-term polynomial to give

$$P(n) = 4.0378n^{4/3} - 27.853n^{5/3} + 52.0859n^2 - 20.7073n^{7/3} + 5.5808n^{8/3}, \quad (29)$$

where $P(n)$ is given in units of $[10^{34} \text{ N/m}^2]$ for n in the density range of $0.1 \text{ fm}^{-3} < n < 3.6 \text{ fm}^{-3}$.

For our non-relativistic equations of state we find that the following equations of state are the best to cover the whole range of densities in a neutron star, and we use:

HZD in the density range of

$$n < 0.000256,$$

BBP-1 in the density range of

$$0.000256 \leq n < 0.003892,$$

BBP-2 in the density range of

$$0.003892 \leq n < n_1,$$

our non-relativistic equations of state in the density range of

$$n_1 \leq n < n_2,$$

AØ-5 in the density range of

$$n_2 \leq n < 3.46,$$

and PS in the density range of

$$n \geq 3.46,$$

where n is given in units of $[\text{fm}^{-3}]$. The coupling points (densities) n_1 and n_2 are here given by

$$n_1 = 0.10, \quad n_2 = 0.80, \quad \text{for } 0\% \text{ protons,}$$

$$n_1 = 0.20, n_2 = 0.85, \text{ for 5\% protons,}$$

$$n_1 = 0.20, n_2 = 0.87, \text{ for 10\% protons,}$$

$$n_1 = 0.35, n_2 = 0.90, \text{ for 15\% protons,}$$

$$n_1 = 0.40, n_2 = 1.05, \text{ for 20\% protons,}$$

$$n_1 = 0.50, n_2 = 1.15, \text{ for 25\% protons.}$$

For our relativistic equation of state, we have coupled the following equations of state: HZD in the density range of

$$n < 0.000256,$$

BBP-1 in the density range of

$$0.000256 \leq n < 0.003892,$$

BBP-2 in the density range of

$$0.003892 \leq n < n_1,$$

our relativistic equations of state in the density range of

$$n_1 \leq n < n_2,$$

and PS in the density range of

$$n \geq n_2,$$

where n is given in units of $[\text{fm}^{-3}]$.

The coupling points (densities) n_1 and n_2 are here given by

$$n_1 = 0.11, n_2 = 0.85, \text{ for 0\% protons,}$$

$$n_1 = 0.14, n_2 = 0.80, \text{ for 5\% protons,}$$

$$n_1 = 0.15, n_2 = 0.85, \text{ for 10\% protons,}$$

$$n_1 = 0.17, n_2 = 0.85, \text{ for 15\% protons,}$$

$$n_1 = 0.18, n_2 = 0.95, \text{ for 20\% protons,}$$

$$n_1 = 0.19, n_2 = 0.93, \text{ for 25\% protons,}$$

$$n_1 = 0.20, n_2 = 0.93, \text{ for 30\% protons,}$$

$$n_1 = 0.21, n_2 = 0.85, \text{ for 35\% protons,}$$

$$n_1 = 0.22, n_2 = 0.90, \text{ for 40\% protons,}$$

$$n_1 = 0.23, n_2 = 0.95, \text{ for 45\% protons,}$$

$$n_1 = 0.24, n_2 = 0.85, \text{ for 50\% protons.}$$

These equations are chosen among 12 published equations of state, and they seem to be the best ones coupled together in the total density range. Total masses, radii, moments of inertia and surface gravitational redshifts are then calculated, and parameterized as functions of the central density n_c . Fig. 2 shows the total mass and fig. 3 the radius versus the central density. Fig. 4 shows the mass versus the radius. Fig. 5 shows the moment of inertia and fig. 6 the gravitational redshift versus the total mass of the star.

IV. DISCUSSIONS AND CONCLUSIONS

From figs. 2–4 we find from the non-relativistic EOS a maximum mass at $M_{\max} = 1.65M_{\odot}$ and the corresponding radius $R = 8.7$ km for pure neutron matter. For a proton fraction of 25% , we find the maximum mass to be $M_{\max} = 1.37M_{\odot}$ with a corresponding radius $R = 6.9$ km. The respective central densities are 1.65 fm^{-3} and 2.33 fm^{-3} (i.e. an order of magnitude higher than normal nuclear matter density).

The maximum mass calculated relativistic equation of state for neutron matter yields $M_{\max} = 2.38M_{\odot}$ with the corresponding radius $R = 12.3$ km. Increasing the proton fraction to 45 % , we get a maximum mass of $M_{\max} = 2.07M_{\odot}$ with the corresponding radius $R = 10.3$ km. The respective central densities are 0.75 fm^{-3} and 0.95 fm^{-3} .

We see that stars calculated with stiff equations of state have greater maximum mass, lower central density (and thicker crust) than stars obtained with soft equations of state.

Observations of binary pulsars give maximum neutron star masses of (Øvergård & Østgaard 1991; Prakash et al. 1988)

$$1.0M_{\odot} < M_{\max} < 2.2M_{\odot},$$

or possibly (Thorsett et al. 1993; Finn 1994; Taylor & Weisberg 1989; Joss & Rappaport 1984; Glendenning 1988)

$$1.3M_{\odot} < M_{\max} < 1.85M_{\odot},$$

At present, no reliable measurements of the radius of a neutron star exist. But general estimates give (Øvergård & Østgaard 1991)

$$R \approx 9\text{km}.$$

If this estimate is close to the true value, then the results from our non-relativistic equations of state may look more reasonable than those from the relativistic one. However, theoretical calculations of the radius of neutron stars can not be confirmed very well by observational data, and are more dependent than the total mass on the low-density equation of state. Moreover the maximum mass occurs at a very high central density, where the relativistic effects certainly prevail.

Data on the nuclear equation of state can, in principle, be obtained from several different sources such as the monopole resonance in nuclei, high energy nuclear collisions, supernovae and neutron stars. Until recently it has, for instance, been assumed that the compression modulus was reasonably well known from the analysis of the giant monopole resonance in nuclei (Blaizot et al. 1976; Blaizot 1980; Treiner et al. 1981). Later, however, these results were questioned by Glendenning (1988) and Brown & Osnes (1985).

Supernova simulations seem to require an equation of state which is too soft to support some observed masses of neutron stars, if sufficient energy shall be released to make the ejection mechanism work (Baron et al. 1985; Woosley & Weaver 1986; Baron et al. 1987).

Supernova explosions can then probably not give a reliable constraint on the nuclear equation of state. Some analyses of high energy nuclear collisions, however, have indicated a moderately stiff or very stiff equation of state (Sano et al. 1985; Stocker & Greiner 1986; Molitoris et al. 1985), although ambiguities have been observed by Gale et al. (1987) and Sharma et al. (1987). Various nuclear data and neutron star masses then seem to favour a rather high compression modulus of $K \sim 300$ MeV (Glendenning 1988; Sharma et al. 1988; Sharma et al. (1989). No definite statements can be made, however, about the equation of state at high densities, except that the neutron star equation of state should probably be moderately stiff to support neutron star masses up to approximately $1.85 M_{\odot}$ (Thorsett et al. 1993).

With the above observations, it seems that our relativistic EOS for neutron matter is too stiff, since the predicted mass $M_{\max} \approx 2.37M_{\odot}$ and radius are larger than the estimated values. We have found that nuclear matter including electrons and muons in beta equilibrium results in a softer EOS. The proton fraction in beta equilibrium is approximately 10-15%.

However, several mechanisms can soften the equation of state for neutron stars considerably. Among these we have mentioned in the introduction kaon and pion condensation. Condensation of the negative charged mesons may be likely to occur if the chemical potential of the mesons becomes equal to the electron chemical potential. If this situation occurs, the proton abundance would increase considerably, possibly up to more than 40% protons, and produce a softer EOS. As mentioned above, when increasing the proton fraction to 45% , the calculated maximum mass is reduced to approximately $2M_{\odot}$ with a corresponding radius of $R \approx 10$ km. This is still slightly higher than the experimental values (Thorsett et al. 93). It should, however, be noted that the gain in condensation energy obtained due to the phase transition to a meson condensate will bring the value of the maximum mass further down.

Further processes which can soften the equation of state are conversion of nucleons to hyperons or a phase transition to quark matter at high densities, which would lower the energy due to an increase of the number of degrees of freedom (Drago et al. 1995). However, for a neutron star to resist the centrifugal forces from very fast rotation, the equation of state should be soft at low and intermediate densities and stiff at high densities, which would not fit very well with the concept of quark matter in hybrid stars (Weber et al. 1991).

From Figs. 2 and 4 we see that the relativistic equations of state give minimum masses $M_{\min} \approx 1.65M_{\odot}$ at central densities around $0.4\text{--}0.5\text{ fm}^{-3}$. This result is almost independent of the proton fraction. This is somewhat higher than the measured masses of neutron stars from ref. (Thorsett et al. 1993). From this we can conclude that the relativistic EOS needs additional softening to fit the data discussed. Even if we assume our relativistic EOS to be a good approximation at normal nuclear matter densities, there have to be softening effects present at $n_c < 0.5\text{ fm}^{-3}$.

In connection to this we stress that our calculation of the EOS is to first order in the reaction matrix G , and we would expect higher-order many-body contributions to also soften the EOS. This was indeed shown in a preliminary study for symmetric nuclear matter by Jiang et al. (1993). Although only a set of higher-order contributions was considered, these authors obtained a softening of the relativistic EOS. Such effects will be studied by us in future works.

From fig. 5 we see that our non-relativistic equations of state give values for the moment of inertia of

$$I(M_{\max}) = 1.05 \times 10^{45} \text{gcm}^2,$$

$$I(M_{\max}) = 0.59 \times 10^{45} \text{gcm}^2,$$

at the maximum mass for pure neutron matter and 25% protons, respectively. The maximum values for moments of inertia obtained from non-relativistic equations of state are

$$I_{\max} = 1.16 \times 10^{45} \text{gcm}^2,$$

$$I_{\max} = 0.65 \times 10^{45} \text{gcm}^2,$$

for pure neutron matter and 25% protons, respectively. The relativistic EOS give

$$I(M_{\max}) = 3.17 \times 10^{45} \text{gcm}^2,$$

$$I(M_{\max}) = 2.29 \times 10^{45} \text{gcm}^2,$$

for pure neutron matter and 45% protons, respectively, and maximum values for moments of inertia are

$$I_{\max} = 3.47 \times 10^{45} \text{gcm}^2,$$

$$I_{\max} = 2.39 \times 10^{45} \text{gcm}^2.$$

These values are not contradictory to observations, and are consistent with the expansion of the Crab nebula and the luminosity and the loss of rotational energy from the Crab pulsar (Mølnvik 1985).

From fig. 6 we see that the gravitational surface redshift is not strongly affected by the different equations of state. This is because the density profiles of the stars are such that their surface gravities are almost the same. A measurement of the redshift can therefore not be used to distinguish between different types of stars or equations of state. It is, however, possible that the slowing down of pulsars and the corresponding glitches can give some information about the internal structure.

In summary, in this work we have calculated the EOS for asymmetric nuclear matter using different proton fractions. Both a non-relativistic and a relativistic Brueckner-Hartree-Fock procedure were employed in order to derive the equation of state, which is the basic input quantity in neutron star calculations since it connects the nuclear physics and the astrophysics. Of importance here is the fact that a relativistic nuclear matter calculation with the Bonn A potential meets the empirical nuclear matter data, a feature not accounted for by non-relativistic calculations. By varying the proton fractions we have estimated certain limits of the neutron star observables from both a relativistic and a non-relativistic approach. From the relativistic approach we obtain a maximum mass of the neutron star that is slightly higher than the empirical ones even when we have a proton fraction close to symmetric nuclear matter. The relativistic effects become important at densities around and higher than the saturation density for nuclear matter, and their main effect is to stiffen the EOS at these densities. This mechanism is due to the fact that the

relativistic effective mass of the nucleon becomes smaller compared to the free mass, an effect which in turn enhances the repulsive spin-orbit term. We obtain a rather low central density which implies that our EOS is too stiff. However, we have discussed several effects that can bring the relativistic results closer to the empirical values. From the non-relativistic EOS we obtain masses and radii closer to empirical values. However, this approach yields an EOS that is too soft to reproduce the empirical data of symmetric nuclear matter. Furthermore, the non-relativistic approach gives substantial proton superconductivity in the interior of the neutron star (De Blasio et al. 1995), and thereby inhibiting the traditional URCA mechanisms.

From our investigation we conclude that our relativistic equations of state are too stiff to reproduce neutron star properties like mass and radius. The existence of exotic states of nuclear matter, such as kaon or pion condensate or quark matter, may explain these discrepancies. However, we may also expect the relativistic EOS to be softened by higher-order many-body contributions.

The other observables like moments of inertia and gravitational redshifts are in good agreement with the accepted values for both the non-relativistic and the relativistic approach.

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FIG. 1. Energy per particle for asymmetric nuclear matter as function of particle density for different proton fractions. Relativistic results are indicated by solid lines and non-relativistic by dashed lines. The increments in proton fractions are 5% for both kinds of results going from pure neutron matter for the top solid and dashed curves to symmetric nuclear matter for the corresponding lowest curves.

FIG. 2. Total mass in units of solar masses $[M_\odot]$ as function of central density for neutron stars. Results obtained from relativistic equations of state are indicated by solid lines, and results obtained from non-relativistic EOS are indicated by dashed lines. The results are shown for different proton fractions.

FIG. 3. Total radius as function of central density for neutron stars. Results obtained from relativistic EOS are indicated by solid lines, and results obtained from non-relativistic EOS are indicated by dashed lines. The results are shown for different proton fractions.

FIG. 4. Mass-radius relations $M(R)$ for neutron stars. For further explanations; see Fig. 1 and Fig. 2.

FIG. 5. Moment of inertia as function of total mass given in $[M_\odot]$ for neutron stars. Results obtained from relativistic EOS are indicated by solid lines, and the results obtained from non-relativistic EOS are indicated by dashed lines.

FIG. 6. Surface gravitational redshift as function of total mass $[M_\odot]$ for neutron stars. For further explanations; see Fig. 5.

Aichelin J., Rosenhauer A., Peilert G., Stocker H. & Greiner W., Phys. Rev. Lett. **58** (1987) 1926
 Bao G., Engvik L., Hjorth-Jensen M., Osnes E. & Østgaard E., Nucl. Phys. **A 575** (1994) 707
 Baron E., Bethe H. A., Brown G. E., Cooperstein J. & Kahana S., Phys. Rev. Lett. **59** (1987) 736
 Baron E., Cooperstein J. & Kahana S., Phys. Rev. Lett. **55** (1985) 126
 Baym G., Bethe H. A. & Pethick C. J., Nucl. Phys. **A175** (1971) 225
 Blaizot J. P., Phys. Rep. **64** (1980) 171
 Blaizot J. P., Gogny D. M.K. & Grammaticos B., Nucl. Phys. **A265** (1976) 315
 Brockmann R., Phys. Rev. **C18** (1978) 1510
 Brockmann R. & Machleidt R., Phys. Rev. **C42** (1990) 1965
 Brown G. E., in proceedings of Perspectives in Nuclear Structure, Niels Bohr Institute, June 14-19, 1993, Nucl. Phys. **A574** (1994) 217c
 Brown G.E., Lee C.-H., Rho M. & Thorsson V., Nucl. Phys. **A567** (1994) 937
 Brown G. E. & Osnes E., Phys. Lett. **B159** (1985) 223
 Brown G. E., Weise W., Baym G. & Speth J., Comments on Nucl. and Part. Physics **17** (1987) 39
 Casares J., Charles P. A. & Naylor T., Nature **355** (1992) 614
 Celenza L. S. & Shakin C. M., *Relativistic Nuclear Physics: Theories of Structure and Scattering*, Vol. 2 of Lecture Notes in Physics (World Scientific, Singapore, 1986)
 Cowley A. P., Crampton D., Hutchings J. B., Remillard R. A. & Penfold J. P., Astrophys. Journ. **272** (1983) 118
 De Blasio F. V., Elgarøy Ø., Engvik L., Hjorth-Jensen M., Lazzari G. & Osnes E. in preparation
 Dickhoff W. H. & Mütter H., Rep. Prog. Phys. **55** (1992) 1947
 Drago A., Tambini U. & Hjorth-Jensen M., ECT* preprint ECT/MAY/95-03 and submitted to Phys. Rev. Lett.
 Engvik L., Hjorth-Jensen M., Osnes E., Bao G. & Østgaard E., Phys. Rev. Lett. **73**, (1994) 2650
 Finn L. S., Phys. Rev. Lett. **73** (1994) 1878
 Gale C., Bertsch G. & Das Gupta S., Phys. Rev. **C35** (1987) 415
 Gies D. R. & Bolton C. T., Astrophys. Journ. **304** (1986) 371
 Glendenning N. K., Phys. Rev. **C37** (1988) 2733
 Glendenning N. K. in Proceedings of the International Summer School on “Structure of Hadrons and Hadronic Matter” (World Scientific, Singapore, 1991), 275
 Haensel P., Zdunik J. L. & Dobaczewski J., Astron. Astrophys. **222** (1989) 353
 Horowitz C. J. & Serot B. D., Nucl. Phys. **A464** (1987) 613
 Itzykson C. & Zuber J.-B. *Quantum Field theory* (McGraw-Hill, New York, 1980)
 Jiang M. F., Machleidt R. & Kuo T. T. S., Phys. Rev. C **47** (1993) 2661
 Joss P. C. & Rappaport S. A., Ann. Rev. Astron. Astrophys. **22** (1984) 537
 Kuo T. T. S. & Ma Z. Y., Phys. Lett. **B127** (1983) 123
 Kuo T. T. S. & Ma Z. Y., in *Nucleon-Nucleon Interaction and Nuclear Many-body Problems* (edited by Wu S. S. and Kuo T. T. S.) (World Scientific, Singapore) p. 178
 Kuo T. T. S., Ma Z. Y. & Vinh Mau R., Phys. Rev. **C33** (1986) 717
 Lattimer J. M., Pethick C. J., Prakash M. & Haensel P., Phys. Rev. Lett. **66** (1991) 2701
 Li G.Q., Machleidt R. & Brockmann R., Phys. Rev. **C46** (1992) 2782
 Lorenz C. P., Ravenhall D. G. & Pethick C. J., Phys. Rev. Lett. **70** (1993) 379
 Machleidt R., Adv. Nucl. Phys. **19** (1989) 189
 Machleidt R. & Li G. Q., Phys. Rep. **242** (1994) 5
 Mc Clintock J.E. & Remillard R. A.,
 Astrophys. Journ. **308** (1986) 110 Mahaux C., Bortignon P. E., Broglia R.A. & Dasso C. H., Phys. Rep. **120** (1985) 1
 Mahaux C. & Sartor R., Phys. Rev. **C 40** (1989) 1833
 Migdal A. B., Saperstein E. E., Troitsky M. A. & Voskresensky D. N., Phys. Rep. **192** (1990) 179
 Mittet R. & Østgaard E., Phys. Rev. C **37** (1988) 1711
 Molitoris J. J., Hahn D. & Stocker H., Nucl. Phys. **A447** (1985) 13c
 Muto T., Takatsuka T., Tamagaki R. & Tatsumi T., Prog. Theor. Phys. Suppl. **112** (1993) 221
 Muto T., Tamagaki R. & Tatsumi T., Prog. Theor. Phys. Suppl. **112** (1993) 159
 Mütter H., Machleidt R. & Brockmann R., Phys. Rev. **C42** (1990) 1981
 Mølnvik T. & Østgaard E., Nucl. Phys. **A437** (1985) 239
 Nikolaus T., Hoch T. & Madland D. G., Phys. Rev. **C46** (1992) 1757
 Pandharipande V. R. & Smith R. A. Nucl. Phys. **A237** (1975) 507
 Pethick C. J. & Ravenhall D. G., Phil. Trans. Roy. Soc. Lond. **A341** (1992) 17
 Pethick C. J., Ravenhall D. G. & C. P. Lorenz C. P., Nucl. Phys. **A584** (1995) 675
 Prakash M., Ainsworth T. L. & Lattimer J. M., Phys. Rev. Lett. **61** (1988) 2518
 Ray L., Hoffman G.W. & Cooker W. R., Phys. Rep. **212** (1991) 223

Rosenhauer A., Staubo E. F., Csernai L. P., Øvergård T. & Østgaard E., Nucl. Phys. **A540** (1992) 630
 Sano M., Gyulassy M., Wakai M. & Kitazoe Y., Phys. Lett. **B156** (1985) 27
 Sharma M. M., Borghols W. T. A., Brandenburg S., Crona S., van der Woude A. & Harakeh M. N., Phys. Rev. **C38** (1988) 2562
 Sharma M. M., Stocker W., Gleissl P. & Brack M., Nucl. Phys. **A504** (1989) 337
 Serot B. D., Rep. Prog. Phys. **55** (1992) 1855
 Serot S. D. & Walecka J. D., Adv. Nucl. Phys. **16** (1986) 1
 Stocker H. & Greiner W., Phys. Rep. **137** (1986) 277
 Takatsuka T., Tamagaki R. & Tatsumi T., Prog. Theor. Phys. Suppl. **112** (1993) 67
 Taylor J. H. & Weisberg J. M., Astrophys. Journ. **345** (1989) 434
 ter Haar B. & Malfliet R., Phys. Rep. **149** (1987) 207
 Thorsett S. E., Arzoumanian Z., McKinnon M. M. & Taylor J. H., Astrophys. J. **405** (1993) L29
 Thompson R. H., Phys. Rev. **D1** (1970) 110
 Thorsson V., Prakash M. & Lattimer J. M., Nucl. Phys. **A572** (1994) 693
 Treiner J., Krevine H., Bohigas O. & Martorell J., Nucl. Phys. **A317** (1981) 253
 Weber F. & Glendenning N. K., “Hadronic Matter and Rotating Relativistic Neutron Stars”, in Proceedings of the Nankai Summer School on Astrophysics and Neutrino physics (World Scientific, Singapore, 1993), 64-183.
 Weber F., Glendenning N. K. & Weigel M. K., Astrophys. Journ. **373** (1991) 579
 Wiringa R.B., Fiks V. & Fabrocini A., Phys. Rev. **C38** (1988) 1010
 Witten E., Phys. Rev. **D 30** (1984) 272
 Woosley S.E. & Weaver T. A., Ann. Rev. Astron. Astropys. **24** (1986) 205
 Øvergård T. & Østgaard E., Can. Journ. Phys. **69** (1991) 8











